

TEMA 7

DISTRIBUCIONES DE PROBABILIDAD CONTINUAS

7.1. DISTRIBUCIÓN UNIFORME $U(a,b)$

- Dominio

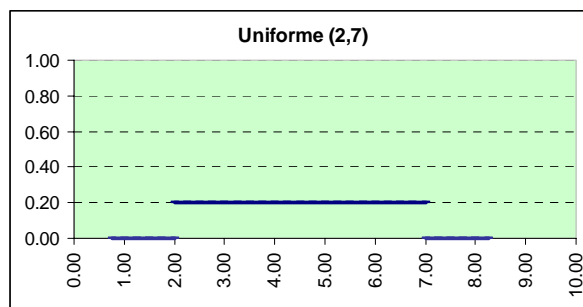
valores entre mínimo (a) y máximo (b)

Ej: El IPC este año puede subir entre el 2 y el 7%, $U(2,7)$

- Distribución de probabilidad

$$f(x) = \frac{1}{b-a} \quad a \leq x \leq b$$

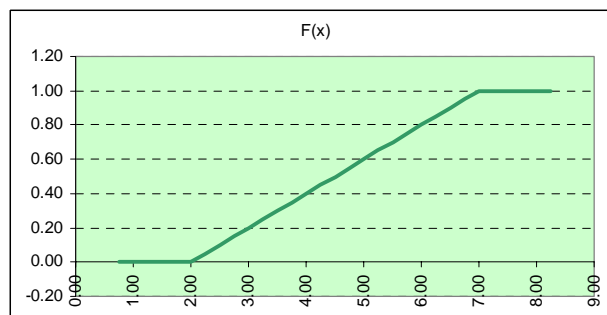
$$\text{Ej: } f(x) = \frac{1}{7-2} = \frac{1}{5} = 0.2 \quad 2 \leq x \leq 7$$



- Función de distribución

$$F(x) = P(\xi \leq x) = \int_a^x \frac{1}{b-a} dx = \frac{1}{b-a} [x]_a^b = \frac{x-a}{b-a} \quad a \leq x \leq b$$

$$F(x) = \frac{x-2}{7-2} = \frac{x-2}{5} \quad 2 \leq x \leq 7$$



- Momentos

$$\begin{cases} E(\xi) = \int_a^b x \frac{1}{b-a} dx = \frac{1}{b-a} \left[\frac{x^2}{2} \right]_a^b = \frac{1}{b-a} \frac{(b^2 - a^2)}{2} = \frac{1}{b-a} \frac{(b-a)(b+a)}{2} = \frac{(a+b)}{2} \\ V(\xi) = \frac{(b-a)^2}{12} \end{cases}$$

$$\text{Ej: } E(\xi) = \frac{(2+7)}{2} = 4.5 \quad V(\xi) = \frac{(7-2)^2}{12} = \frac{25}{12}$$

- Función característica

$$\varphi(t) = \frac{e^{itb} - e^{ita}}{it(b-a)}$$

- Ejemplo: El volumen de ventas de una empresa está entre 380 y 1200 miles de euros:

- Probabilidad de ventas mayor que 1 millón

$$\xi \rightarrow U(380000, 1200000)$$

$$f(x) = \frac{1}{1200000 - 380000} \quad 380000 \leq x \leq 1200000$$

$$F(x) = \frac{x - 380000}{1200000 - 380000} = \frac{x - 380000}{820000} \quad 380000 \leq x \leq 1200000$$

$$P(\xi > 1000000) = 1 - P(\xi \leq 1000000) = 1 - F(1000000) = \frac{1000000 - 380000}{820000} = 0.2439$$

- Esperanza y desviación típica

$$E(\xi) = \frac{(380000 + 1200000)}{2} = 790000$$

$$V(\xi) = \frac{(1200000 - 380000)^2}{12} = 56033.33 * 10^6$$

$$\sqrt{V(\xi)} = 236.71 * 10^3$$

- Ejemplo: Si una variable se distribuye uniformemente con una media de 8 y una desviación típica de 3, cuál es la probabilidad de que $\xi > 9$:

- $\xi \rightarrow U(a, b)$

$$\begin{cases} E(\xi) = \frac{(a+b)}{2} = 8 & \Rightarrow a+b = 16 \\ V(\xi) = \frac{(b-a)^2}{12} = 9 & \Rightarrow b-a = \sqrt{12*9} = \sqrt{108} \end{cases}$$

Resolviendo el sistema de dos ecuaciones :

$$2b = 16 + \sqrt{108} \Rightarrow b = \frac{16 + \sqrt{108}}{2} = 8 + \sqrt{27}$$

$$a = 16 - b = 16 - 8 - \sqrt{27} = 8 - \sqrt{27}$$

$$U(8 - \sqrt{27}, 8 + \sqrt{27})$$

$$\circ \quad F(x) = \frac{x - (8 - \sqrt{27})}{(8 + \sqrt{27}) - (8 - \sqrt{27})} = \frac{x - 8 + \sqrt{27}}{2\sqrt{27}}$$

$$P(\xi > 9) = 1 - P(\xi \leq 9) = 1 - F(9) = 1 - \frac{9 - 8 + \sqrt{27}}{2\sqrt{27}} = 0.4038$$

7.2. DISTRIBUCIÓN NORMAL $N(\mu, \sigma)$

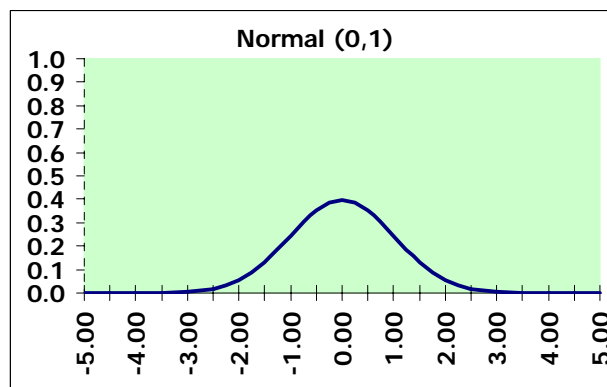
7.2.1. DISTRIBUCIÓN NORMAL $N(0, 1)$

- Dominio

valores entre $-\infty$ y $+\infty$, toda la recta real

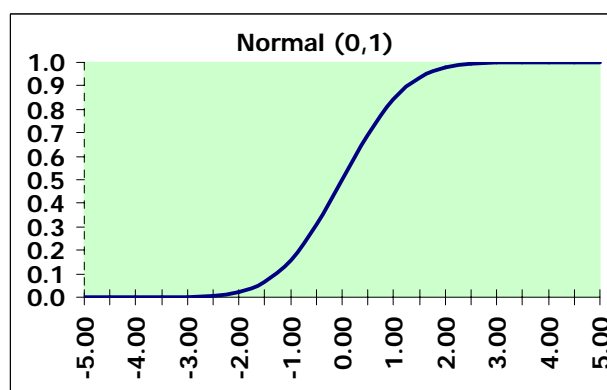
- Distribución de probabilidad

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad -\infty < x < \infty \quad (\text{Campana de Gauss, 1809})$$



- Función de distribución

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{x^2}{2}} dx \quad (\text{Tablas})$$



- Función característica

$$\varphi(t) = e^{-\frac{1}{2}t^2}$$

- Momentos

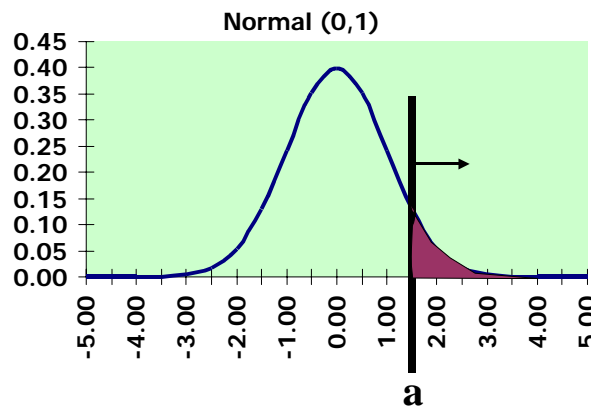
$$\begin{cases} E(\xi) = \frac{1}{i} \left| -te^{-\frac{1}{2}t^2} \right|_{t=0} = \frac{1}{i} |0| = 0 \\ V(\xi) = \frac{1}{i^2} \left| t^2 e^{-\frac{1}{2}t^2} - e^{-\frac{1}{2}t^2} \right|_{t=0} - (0)^2 = \frac{1}{i^2} |0-1| = \frac{1}{-1} |0-1| = 1 \end{cases}$$

$$\mu_{2k} = \frac{(2k)!}{2^k k!} \quad \text{momentos de orden par}$$

$$\mu_{2k+1} = 0 \quad \text{momentos de orden impar}$$

7.2.2. MANEJO DE TABLAS. CÁLCULO DE PROBABILIDADES

- Las tablas del libro incluyen $P(\xi \geq a)$. Puede haber otros libros con otras tablas: hay que fijarse en el dibujo



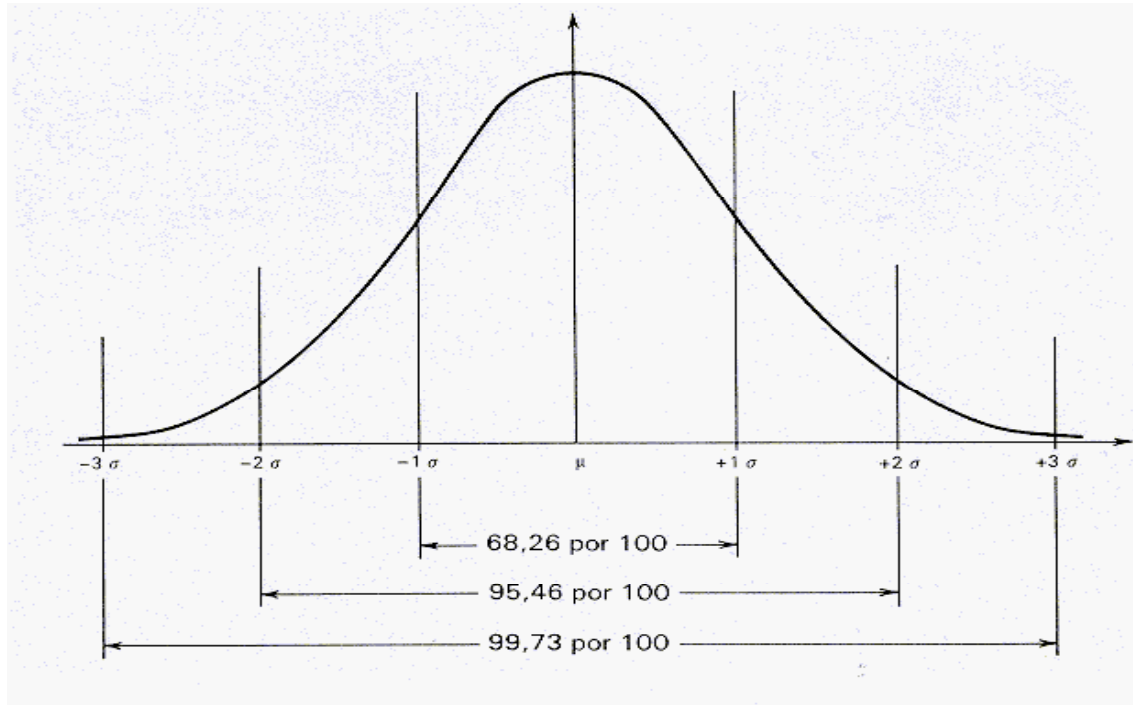
- Las tablas incluyen:
 - En los ejes el valor de x
 - Eje vertical o filas: el valor entero y el primer decimal
 - Eje horizontal o columnas: el segundo decimal
 - Dentro de la tabla: el valor de la probabilidad
- Ejemplos 1:
 - $P(\xi \geq 0) = P(\xi \geq 0.00) = P(\text{fila } 0.0, \text{ columna } 0.00) = 0.5000$
 - $P(\xi \geq 1) = P(\xi \geq 1.00) = P(\text{fila } 1.0, \text{ columna } 0.00) = 0.1587$
 - $P(\xi \geq 2) = P(\xi \geq 2.00) = P(\text{fila } 2.0, \text{ columna } 0.00) = 0.0228$
 - $P(\xi \geq 3) = P(\xi \geq 3.00) = P(\text{fila } 3.0, \text{ columna } 0.00) = 0.0014$
 - $P(\xi \geq 4) = P(\xi \geq 4.00) = P(\text{fila } 4.0, \text{ columna } 0.00) = 0.0000$
- Ejemplos 2:
 - $P(\xi < 1) = 1 - P(\xi \geq 1.00) = 1 - P(\text{fila } 1.0, \text{ columna } 0.00) = 1 - 0.1587 = 0.8413$
 - $P(\xi < 2) = 1 - P(\xi \geq 2.00) = 1 - P(\text{fila } 2.0, \text{ columna } 0.00) = 1 - 0.0228 = 0.9772$
 - $P(\xi < 3) = 1 - P(\xi \geq 3.00) = 1 - P(\text{fila } 3.0, \text{ columna } 0.00) = 1 - 0.0014 = 0.9986$
 - $P(\xi < 4) = 1 - P(\xi \geq 4.00) = 1 - P(\text{fila } 4.0, \text{ columna } 0.00) = 1 - 0.0000 = 1.0000$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

- Ejemplos 3: $P(\xi \leq -a) = P(\xi \geq a)$ por simetría
 - $P(\xi \leq -1) = P(\xi \geq 1.00) = P(\text{fila 1.0, columna 0.00}) = 0.1587$
 - $P(\xi \leq -2) = P(\xi \geq 2.00) = P(\text{fila 2.0, columna 0.00}) = 0.0228$
 - $P(\xi \leq -3) = P(\xi \geq 3.00) = P(\text{fila 3.0, columna 0.00}) = 0.0014$
 - $P(\xi \leq -4) = P(\xi \geq 4.00) = P(\text{fila 4.0, columna 0.00}) = 0.0000$
- Ejemplos 4:
 - $P(\xi \geq 0.56) = P(\text{fila 0.5, columna 0.06}) = 0.2877$
 - $P(\xi \leq -0.24) = P(\xi \geq 0.24) = P(\text{fila 0.2, columna 0.04}) = 0.4052$
 - $P(\xi \leq 1.36) = 1 - P(\xi > 1.36) = 1 - P(\text{fila 1.3, columna 0.06}) = 1 - 0.0869 = 0.9131$
 - $P(\xi \geq -2.50) = 1 - P(\xi \leq -2.50) = 1 - P(\xi \geq 2.50) = P(\text{fila 2.5, columna 0.00}) = 1 - 0.0062 = 0.9938$
- Ejemplos 5: $P(\xi = 1) = P(1 \leq \xi \leq 1) = 0$

- Ejemplos 6:

- $P(-1 \leq \xi \leq 1) = P(\xi \leq 1) - P(\xi < -1) = 0.8413 - 0.1587 = 0.6826$
 - $P(-2 \leq \xi \leq 2) = P(\xi \leq 2) - P(\xi < -2) = 0.9772 - 0.0228 = 0.9544$
 - $P(-3 \leq \xi \leq 3) = P(\xi \leq 3) - P(\xi < -3) = 0.9986 - 0.0014 = 0.9972$



- Ejemplos 7:

- $P(0.30 < \xi \leq 2.89) = P(\xi \leq 2.89) - P(\xi \leq 0.30) = (1 - 0.0019) - (1 - 0.3821) = 0.3802$
 - $P(-0.70 \leq \xi \leq -0.15) = P(\xi \leq -0.15) - P(\xi < -0.70) = 0.4404 - 0.2420 = 0.1984$
 - $P(-1.12 \leq \xi \leq 1.63) = P(\xi \leq 1.63) - P(\xi < -1.12) = (1 - 0.0516) - 0.1314 = 0.8170$

- Ejemplos 8:

- $P(\xi \geq a) = 0.0192 \rightarrow a = 2.07$
 - $P(\xi \leq a) = 0.8485 \rightarrow P(\xi \geq a) = 0.1515 \rightarrow a = 1.03$
 - $P(\xi \leq a) = 0.2389 \rightarrow P(\xi \geq -a) = 0.2389 \rightarrow -a = 0.71 \rightarrow a = -0.71$
 - $P(\xi \geq a) = 0.7324 \rightarrow P(\xi \leq a) = P(\xi \geq -a) = 0.2676 \rightarrow -a = 0.62 \rightarrow a = -0.62$

- Ejemplo 9:

- $P(-0.68 \leq \xi \leq a) = 0.7289 \rightarrow P(\xi \leq a) - P(\xi \leq -0.68) = 0.7289$
 $\rightarrow P(\xi \leq a) = P(\xi \leq -0.68) + 0.7289$
 $\rightarrow P(\xi \leq a) = P(\xi \geq 0.68) + 0.7289$
 $\rightarrow P(\xi \leq a) = 0.2483 + 0.7289$
 $\rightarrow P(\xi \leq a) = 0.9772$
 $\rightarrow P(\xi \geq a) = 0.0228$
 $\rightarrow a = 2$

7.2.3. DISTRIBUCIÓN NORMAL $N(\mu, \sigma)$

- Si:
 - $-\infty < \mu < \infty, \sigma > 0$
 - $\eta = \mu + \sigma \xi$
 - $\xi = N(0,1)$

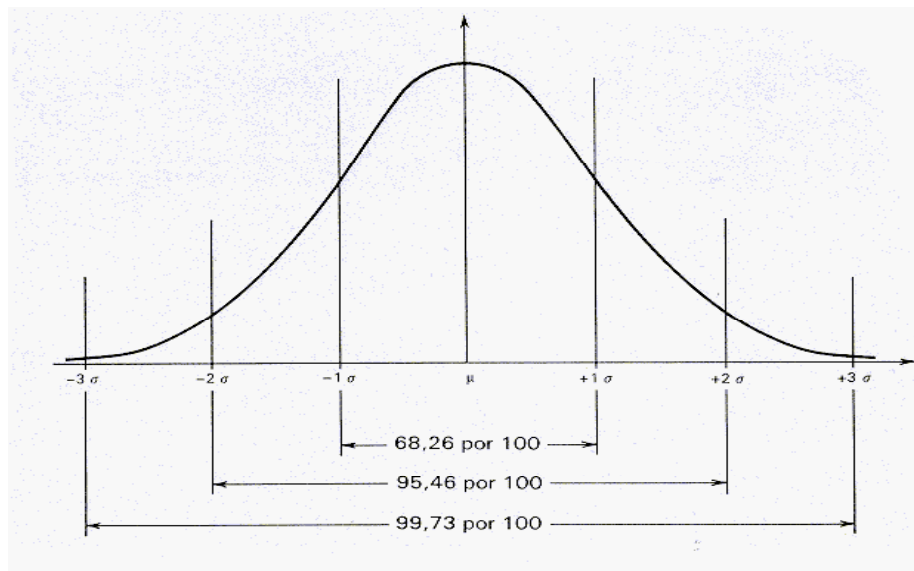
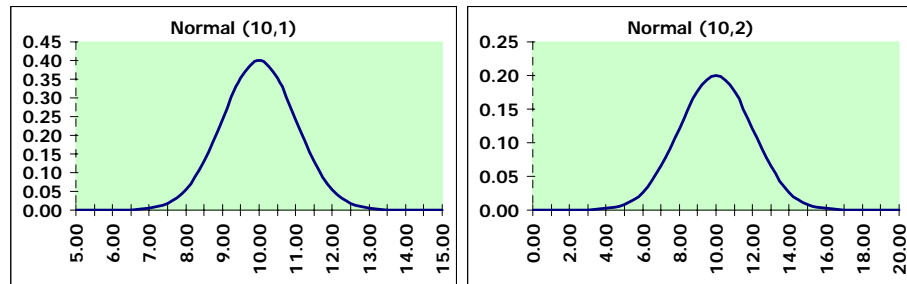
entonces

- $\eta = N(\mu, \sigma)$
- Dominio

valores entre $-\infty$ y $+\infty$, toda la recta real
- Distribución de probabilidad

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$$

Misma forma todas...



Si $N(10,2)$:

$$\begin{array}{ccccccc}
 4 & 6 & 8 & 10 & 12 & 14 & 16 \\
 (10-3 \cdot 2) & & & & & & (10+3 \cdot 2)
 \end{array}$$

- Función de distribución

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \quad (\text{Tablas después de tipificar a } N(0,1))$$

- Momentos

$$\begin{cases} E(\eta) = E(\mu + \sigma\xi) = \mu + \sigma E(\xi) = \mu + \sigma \cdot 0 = \mu \\ V(\eta) = V(\mu + \sigma\xi) = V(\mu) + \sigma^2 V(\xi) = 0 + \sigma^2 \cdot 1 = \sigma^2 \end{cases}$$

$$\gamma_1 = \frac{\mu_3}{\sigma^3} = 0$$

$$\gamma_2 = \frac{\mu_4}{\sigma^4} - 3 = \frac{3\sigma^4}{\sigma^4} - 3 = 0$$

$$\mu_{2k+1} = 0$$

- Función característica

$$\varphi(t) = e^{it\mu - \frac{1}{2}t^2\sigma^2}$$

- Si:

$$\circ \quad \eta = \mu + \sigma\xi$$

entonces

$$\circ \quad \xi = z = \frac{\eta - \mu}{\sigma} \quad \text{tipificación}$$

- Problema 1: $\eta \rightarrow N(5,2)$

$$\circ \quad P(\eta \geq 8) = P\left(\xi \geq \frac{8-5}{\sqrt{2}}\right) = P(\xi \geq 1.5) = 0.0668$$

- Problema 2: $\eta \rightarrow N(-5,10)$

$$\begin{aligned} \circ \quad P(|\eta| < 5.1) &= P(-5.1 < \eta < 5.1) = P\left(\xi < \frac{5.1 - (-5)}{\sqrt{10}}\right) - P\left(\xi < \frac{(-5.1) - (-5)}{\sqrt{10}}\right) = \\ &= P(\xi \leq \underline{1.01}) - P(\xi \leq \underline{-0.01}) = [1 - P(z \geq \underline{1.01})] - [P(z > \underline{-0.01})] = \\ &= (1 - 0.1563) - 0.4960 = 0.3477 \end{aligned}$$

- Problema 3: $\eta \rightarrow N(2,0.1)$

$$\circ \quad P(a < \eta \leq 2.2) = 0.65$$

$$\circ \quad P(\eta \leq 2.2) - P(\eta \leq a) = P\left(\xi < \frac{2.2 - 2}{\sqrt{0.1}}\right) - P\left(\xi < \frac{a - 2}{\sqrt{0.1}}\right) =$$

$$= P(\xi \leq 2) - P\left(\xi \leq \frac{a - 2}{\sqrt{0.1}}\right) = 0.65$$

$$\circ \quad P(\xi \leq 2) - 0.65 = P\left(\xi \leq \frac{a - 2}{\sqrt{0.1}}\right)$$

$$\circ \quad [1 - P(\xi \geq 2)] - 0.65 = P\left(\xi \leq \frac{a - 2}{\sqrt{0.1}}\right)$$

- $[1-0.0228]-0.65 = P(\xi \leq \frac{a-2}{0.1})$
- $0.3272 = P(\xi \leq \frac{a-2}{0.1})$
- $0.3272 = P(\xi \geq -\frac{a-2}{0.1})$
- $-\frac{a-2}{0.1} = 0.445$
- $a = 2 - 0.445(0.1) = 1.9555$
- Propiedad aditiva
 - Si $\varpi = b + a_1\eta_1 + a_2\eta_2 + \dots + a_n\eta_n = b + \sum_{j=1}^n a_j\eta_j$

donde $\eta_j \rightarrow N(\mu_j, \sigma_j)$ e independientes

entonces:

$$\varpi = N\left(b + \sum_{j=1}^n a_j\mu_j, \sqrt{\sum_{j=1}^n a_j^2\sigma_j^2}\right)$$

$$E(\varpi) = E\left(b + \sum_{j=1}^n a_j\eta_j\right) = E(b) + E\left(\sum_{j=1}^n a_j\eta_j\right) = b + \sum_{j=1}^n a_j E(\eta_j) = b + \sum_{j=1}^n a_j\mu_j$$

$$V(\varpi) = V\left(b + \sum_{j=1}^n a_j\eta_j\right) = V(b) + V\left(\sum_{j=1}^n a_j\eta_j\right) = 0 + \sum_{j=1}^n a_j^2 V(\eta_j) = \sum_{j=1}^n a_j^2 \sigma_j^2$$

$$\varphi_{\varpi}(t) = e^{it\left(b + \sum_{j=1}^n a_j\mu_j\right) - \frac{1}{2}t^2\left(\sum_{j=1}^n a_j^2\sigma_j^2\right)}$$

- Casos Particulares
 - Distribución de la SUMA de normales
 - $\eta_j \rightarrow N(\mu_j, \sigma_j)$ e independientes

$$\varpi = \sum_{j=1}^n \eta_j$$

$$\varpi = N\left(\sum_{j=1}^n \mu_j, \sqrt{\sum_{j=1}^n \sigma_j^2}\right)$$

- $\eta_j \rightarrow N(\mu, \sigma)$ e independientes, iid

$$\varpi = \sum_{j=1}^n \eta_j$$

$$\varpi = N\left(n\mu, \sqrt{n\sigma^2}\right) = N\left(n\mu, \sigma\sqrt{n}\right)$$

- Distribución de la MEDIA de normales
 - $\eta_j \rightarrow N(\mu_j, \sigma_j)$ e independientes

$$\tau = \frac{\sum_{j=1}^n \eta_j}{n}$$

$$\tau = N\left(\frac{\sum_{j=1}^n \mu_j}{n}, \sqrt{\frac{\sum_{j=1}^n \sigma_j^2}{n}}\right)$$

- $\eta_j \rightarrow N(\mu, \sigma)$ e independientes, iid

$$\tau = \frac{\sum_{j=1}^n \eta_j}{n}$$

$$\tau = N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

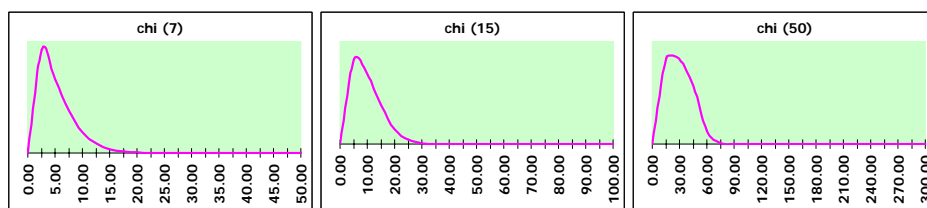
- Problema 1: Si cada bote de cerveza sigue una distribución $\eta_i \rightarrow N(30, 2)$
 - Probabilidad de que un bote tenga más de 33 cl
 $\eta \rightarrow N(30, 2)$
 $P(\eta > 33) = P(\xi \geq \frac{33-30}{2}) = P(\xi \geq 1.5) = 0.0668$
 - Probabilidad de que un envase de 6 latas contenga menos de 1.75 litros
 $\varpi = N(6 * 30, \sqrt{6 * 2^2}) = N(6 * 30, 2\sqrt{6}) = N(180, \sqrt{24})$
 $P(\varpi < 175) = P(\xi \leq \frac{175-180}{\sqrt{24}}) = P(\xi \leq -1.0206) = P(\xi \geq 1.0206) = 0.1539$

7.3. DISTRIBUCIONES DERIVADAS DE LA NORMAL

7.3.1. DISTRIBUCIÓN χ^2 DE PEARSON

χ^2 CENTRAL

- Si $\eta = \xi_1^2 + \xi_2^2 + \dots + \xi_n^2$
 donde $\xi_i \rightarrow N(0, 1)$ iid



entonces $\eta \rightarrow \chi^2(n)$ central, donde $n \equiv$ grados de libertad

- $E(\chi^2(n))=n$
 $V(\chi^2(n))=2n$
- Propiedad Aditiva
 $\Sigma \chi^2(n_j) = \chi^2(\Sigma n_j)$
- Tablas para calcular probabilidades $P(\chi^2(n) \geq a)$. Los valores de x están dentro de la tabla, la n en las filas y la probabilidad en las columnas.

	Probabilidad										
Grados de Libertad	0.990	0.975	0.950	0.900	0.750	0.500	0.250	0.100	0.050	0.025	0.010
1	0.000	0.001	0.004	0.016	0.102	0.455	1.323	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	0.575	1.386	2.773	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	1.213	2.366	4.108	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	1.923	3.357	5.385	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	2.675	4.351	6.626	9.236	11.070	12.832	15.086
6	0.872	1.237	1.635	2.204	3.455	5.348	7.841	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	4.255	6.346	9.037	12.017	14.067	16.013	18.475
8	1.647	2.180	2.733	3.490	5.071	7.344	10.219	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	5.899	8.343	11.389	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	6.737	9.342	12.549	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	7.584	10.341	13.701	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	8.438	11.340	14.845	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.041	9.299	12.340	15.984	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	10.165	13.339	17.117	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	11.037	14.339	18.245	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	11.912	15.338	19.369	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	12.792	16.338	20.489	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	13.675	17.338	21.605	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	14.562	18.338	22.718	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	15.452	19.337	23.828	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	16.344	20.337	24.935	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	17.240	21.337	26.039	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	18.137	22.337	27.141	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	19.037	23.337	28.241	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	19.939	24.337	29.339	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	20.843	25.336	30.435	35.563	38.885	41.923	45.642
27	12.878	14.573	16.151	18.114	21.749	26.336	31.528	36.741	40.113	43.195	46.963
28	13.565	15.308	16.928	18.939	22.657	27.336	32.620	37.916	41.337	44.461	48.278
29	14.256	16.047	17.708	19.768	23.567	28.336	33.711	39.087	42.557	45.722	49.588
30	14.953	16.791	18.493	20.599	24.478	29.336	34.800	40.256	43.773	46.979	50.892
40	22.164	24.433	26.509	29.051	33.660	39.335	45.616	51.805	55.758	59.342	63.691
50	29.707	32.357	34.764	37.689	42.942	49.335	56.334	63.167	67.505	71.420	76.154
60	37.485	40.482	43.188	46.459	52.294	59.335	66.981	74.397	79.082	83.298	88.379
70	45.442	48.758	51.739	55.329	61.698	69.334	77.577	85.527	90.531	95.023	100.425
80	53.540	57.153	60.391	64.278	71.145	79.334	88.130	96.578	101.879	106.629	112.329
90	61.754	65.647	69.126	73.291	80.625	89.334	98.650	107.565	113.145	118.136	124.116
100	70.065	74.222	77.929	82.358	90.133	99.334	109.141	118.498	124.342	129.561	135.807

- Ejemplo

- $P(3.490 \leq \chi^2(8) \leq 10.219) = P(\chi^2(8) \leq 10.219) - P(\chi^2(8) \leq 3.490)$
 $= [1 - P(\chi^2(8) \geq 10.219)] - [1 - P(\chi^2(8) \geq 3.490)]$
 $= [1 - 0.25] - [1 - 0.90] = 0.75 - 0.10 = 0.65$
- $P(\chi^2(7) \geq a) = 0.10 \rightarrow a = 12.0170$
- $P(\chi^2(11) \leq a) = 0.10 \rightarrow P(\chi^2(11) \geq a) = 0.90 \rightarrow a = 5.5780$

7.3.2. DISTRIBUCIÓN t DE STUDENT

t CENTRAL

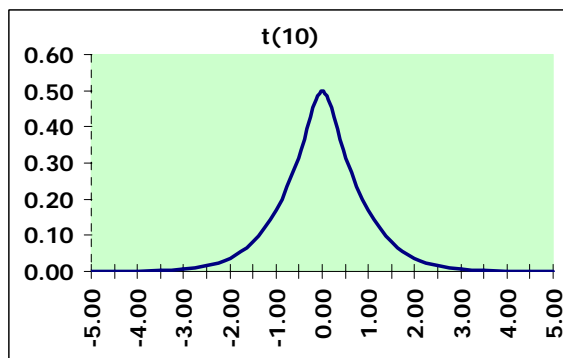
- Sean $(n+1)$ $N(0, \sigma)$ iid

entonces:

$$t(n) = \frac{\eta}{\sqrt{\frac{\eta_1^2 + \eta_2^2 + \dots + \eta_n^2}{n}}}$$

donde $n \equiv$ grados de libertad

- $f(x)$: simétrica, un poco más chata que la normal, tendiendo a ella cuando $n \rightarrow \infty$



- No depende de σ , y se usa para estimación cuando la muestra es pequeña
- $E(t(n)) = 0$

$$V(t(n)) = \frac{n}{n-2}$$

- Ejemplo de cálculo de probabilidades $P(t(n) \geq a)$ utilizando tablas. Los valores de x están dentro de la tabla, la n en las filas y la probabilidad en las columnas.
 - $P(-1.156 \leq t(5) \leq 1.156) = P(t(5) \leq 1.156) - P(t(5) \leq -1.156)$
 $= [1 - 0.15] - [0.15] = 0.70$
 - $P(t(10) \leq a) = 0.975 \rightarrow a = 2.2281$
 - $P(t(1000) \geq a) = 0.05 \rightarrow P(N(0,1) \geq a) = 0.05 \rightarrow a = 1.645$

	Probabilidad									
Grados de Libertad	0.400	0.250	0.200	0.150	0.100	0.050	0.025	0.020	0.010	0.005
1	0.3249	1.0000	1.3764	1.9626	3.0777	6.3137	12.7062	15.8945	31.8210	63.6559
2	0.2887	0.8165	1.0607	1.3862	1.8856	2.9200	4.3027	4.8487	6.9645	9.9250
3	0.2767	0.7649	0.9785	1.2498	1.6377	2.3534	3.1824	3.4819	4.5407	5.8408
4	0.2707	0.7407	0.9410	1.1896	1.5332	2.1318	2.7765	2.9985	3.7469	4.6041
5	0.2672	0.7267	0.9195	1.1558	1.4759	2.0150	2.5706	2.7565	3.3649	4.0321
6	0.2648	0.7176	0.9057	1.1342	1.4398	1.9432	2.4469	2.6122	3.1427	3.7074
7	0.2632	0.7111	0.8960	1.1192	1.4149	1.8946	2.3646	2.5168	2.9979	3.4995
8	0.2619	0.7064	0.8889	1.1081	1.3968	1.8595	2.3060	2.4490	2.8965	3.3554
9	0.2610	0.7027	0.8834	1.0997	1.3830	1.8331	2.2622	2.3984	2.8214	3.2498
10	0.2602	0.6998	0.8791	1.0931	1.3722	1.8125	2.2281	2.3593	2.7638	3.1693
11	0.2596	0.6974	0.8755	1.0877	1.3634	1.7959	2.2010	2.3281	2.7181	3.1058
12	0.2590	0.6955	0.8726	1.0832	1.3562	1.7823	2.1788	2.3027	2.6810	3.0545
13	0.2586	0.6938	0.8702	1.0795	1.3502	1.7709	2.1604	2.2816	2.6503	3.0123
14	0.2582	0.6924	0.8681	1.0763	1.3450	1.7613	2.1448	2.2638	2.6245	2.9768
15	0.2579	0.6912	0.8662	1.0735	1.3406	1.7531	2.1315	2.2485	2.6025	2.9467
16	0.2576	0.6901	0.8647	1.0711	1.3368	1.7459	2.1199	2.2354	2.5835	2.9208
17	0.2573	0.6892	0.8633	1.0690	1.3334	1.7396	2.1098	2.2238	2.5669	2.8982
18	0.2571	0.6884	0.8620	1.0672	1.3304	1.7341	2.1009	2.2137	2.5524	2.8784
19	0.2569	0.6876	0.8610	1.0655	1.3277	1.7291	2.0930	2.2047	2.5395	2.8609
20	0.2567	0.6870	0.8600	1.0640	1.3253	1.7247	2.0860	2.1967	2.5280	2.8453
21	0.2566	0.6864	0.8591	1.0627	1.3232	1.7207	2.0796	2.1894	2.5176	2.8314
22	0.2564	0.6858	0.8583	1.0614	1.3212	1.7171	2.0739	2.1829	2.5083	2.8188
23	0.2563	0.6853	0.8575	1.0603	1.3195	1.7139	2.0687	2.1770	2.4999	2.8073
24	0.2562	0.6848	0.8569	1.0593	1.3178	1.7109	2.0639	2.1715	2.4922	2.7970
25	0.2561	0.6844	0.8562	1.0584	1.3163	1.7081	2.0595	2.1666	2.4851	2.7874
26	0.2560	0.6840	0.8557	1.0575	1.3150	1.7056	2.0555	2.1620	2.4786	2.7787
27	0.2559	0.6837	0.8551	1.0567	1.3137	1.7033	2.0518	2.1578	2.4727	2.7707
28	0.2558	0.6834	0.8546	1.0560	1.3125	1.7011	2.0484	2.1539	2.4671	2.7633
29	0.2557	0.6830	0.8542	1.0553	1.3114	1.6991	2.0452	2.1503	2.4620	2.7564
30	0.2556	0.6828	0.8538	1.0547	1.3104	1.6973	2.0423	2.1470	2.4573	2.7500
35	0.2553	0.6816	0.8520	1.0520	1.3062	1.6896	2.0301	2.1332	2.4377	2.7238
40	0.2550	0.6807	0.8507	1.0500	1.3031	1.6839	2.0211	2.1229	2.4233	2.7045
45	0.2549	0.6800	0.8497	1.0485	1.3007	1.6794	2.0141	2.1150	2.4121	2.6896
50	0.2547	0.6794	0.8489	1.0473	1.2987	1.6759	2.0086	2.1087	2.4033	2.6778
60	0.2545	0.6786	0.8477	1.0455	1.2958	1.6706	2.0003	2.0994	2.3901	2.6603
70	0.2543	0.6780	0.8468	1.0442	1.2938	1.6669	1.9944	2.0927	2.3808	2.6479
80	0.2542	0.6776	0.8461	1.0432	1.2922	1.6641	1.9901	2.0878	2.3739	2.6387
90	0.2541	0.6772	0.8456	1.0424	1.2910	1.6620	1.9867	2.0839	2.3685	2.6316
100	0.2540	0.6770	0.8452	1.0418	1.2901	1.6602	1.9840	2.0809	2.3642	2.6259
120	0.2539	0.6765	0.8446	1.0409	1.2886	1.6576	1.9799	2.0763	2.3578	2.6174
150	0.2538	0.6761	0.8440	1.0400	1.2872	1.6551	1.9759	2.0718	2.3515	2.6090
200	0.2537	0.6757	0.8434	1.0391	1.2858	1.6525	1.9719	2.0672	2.3451	2.6006
300	0.2536	0.6753	0.8428	1.0382	1.2844	1.6499	1.9679	2.0627	2.3388	2.5923
∞	0.2533	0.6745	0.8416	1.0364	1.2816	1.6449	1.9600	2.0537	2.3263	2.5758